hep-th/9511001
October 1995

Quantization of 2D Abelian Gauge Theory without the Kinetic Term of Gauge Field as Anomalous Gauge Theory

M.Koseki [a] ,R.Kuriki [b]

Graduate School of Science and Technology, Niigata University, Niigata, 950-21, Japan
Department of Physics, Tokyo Institute of Technology, Tokyo 152, Japan

Abstract

The massless Schwinger model without the kinetic term of gauge field has gauge anomaly. We quantize the model as an anomalous gauge theory in the most general class of gauge conditions. We show that the gauge field becomes a dynamical variable because of gauge anomaly.

[a]E-mail address: koseki@muse.hep.sc.niigata-u.ac.jp
[b]E-mail address: r-kuriki@phys.titech.ac.jp

1 Introduction

Two-dimensional field theories coupled to gauge fields without kinetic terms have recently been investigated in several context [1-4]. There are two motivations to study such gauge theory.

One of them is the expectation that the models may provide realizations of the GKO coset construction of conformal field theories[5]. The Schwinger model without kinetic term of gauge field corresponds to the $U(1)/U(1)$ coset model in the language of GKO coset construction. Actually, Itoi and Mukaida studied the model [6], and they asserted that the model was a topological field theory [7].

Another motivation to study the gauge theory without a kinetic term is the expectation that the analysis of such gauge theory will help us to investigate non-critical string theories. Actually, Polyakov studied such a gauge theory as a toy model of two-dimensional gravity [2].

The symmetries in the Schwinger model without kinetic term of gauge field is completely different from one in the usual Schwinger model. Moreover, the Schwinger model without a kinetic term of gauge field has gauge anomaly, but there is no gauge anomaly in the usual Schwinger model [8]. Therefore, one can treat this model as an anomalous gauge theory.

There are two purposes of this paper. The one of them is to formulate the Schwinger model without a kinetic term of gauge field as an anomalous gauge theory in the most general class of gauge fixing conditions. The other is to show that a degree of freedom of gauge field becomes a dynamical variable because of gauge anomaly, though one can eliminate the all degree of freedom of gauge field by local symmetries at classical level.

The strategy to achieve our aim is as follows. To avoid the confusion between this model and the usual Schwinger model, we show that gauge anomaly exists in this model in Appendix A. In Section 2, the Hamiltonian formalism developed by Batalin and Fradkin (BF formalism) is applied to convert the constraints effectively from second class to first class. A generally BRST quantization scheme by the extended phase space of Batalin, Fradkin and Vilkovisky (BFV) is constructed. In Section 3, we investigate the theory under two types of gauge fixing conditions. In the light-cone gauge, we show that the degree of freedom of gauge field becomes a dynamical variable at quantum level. Conclusion are given in Section 4.

2 Symmetrization and BRST invariant effective action in the most general class of gauges

In this section, first, we investigate properties of constraints in this model. The constraints belong to the first class at the classical level, but these constraints belong to the second class at quantum level because of gauge anomaly. We convert them into first class by introducing an extra degree of freedom. Finally, the converted system is quantized in the extended phase space, and a BRST invariant effective action is constructed.

First, we summarize the properties of the symmetry in the Schwinger model without the kinetic term of gauge field. The Lagrangian density of the Schwinger model without the kinetic term of gauge field is given by

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\gamma^{\mu}\left(i\partial_{\mu}+A_{\mu}\right)\psi \\ &= \psi_{+}^{\dagger}\left(i\partial_{-}+A_{-}\right)\psi_{+}+\psi_{-}^{\dagger}\left(i\partial_{+}+A_{+}\right)\psi_{-},\end{aligned} \tag{2.1}$$

where ψ and A_{μ} $(\mu=0,1)$ are Dirac field and $U(1)$ gauge field [‡], respectively. This Lagrangian density has $U(1)_V\times U(1)_A$ local symmetries at classical level[10]. Although the model has two local symmetries at classical level, no regularization of it can preserve both the symmetries simultaneously. Not all of these symmetries can survive because of anomalies in ordinary configuration space.

The equal time super-Poisson bracket among these variables are given by

$$\begin{aligned}\left\{A_{\pm}(x,t),\pi_{\pm}^{A}(y,t)\right\} &= \delta(x-y), \\ \left\{\psi_{\pm}(x,t),\psi_{\pm}^{\dagger}(y,t)\right\} &= -i\delta(x-y).\end{aligned} \tag{2.2}$$

In the canonical description, the local symmetries manifest themselves as first class constraints according to Dirac 's classification [11]. Denoting the canonical momenta for $A_{\pm}$ by $\pi_{\pm}^{A}$, we write the primary constraints as

$$\pi_{\pm}^{A}\approx 0, \tag{2.3}$$

[‡]We use the following conventions in two dimensions;

$$\gamma^0=\sigma^1,\gamma^1=i\sigma^2,\gamma^5=\gamma^0\gamma^1,\partial_{\pm}=\partial_0\pm\partial_1,A_{\pm}=A_0\pm A_1$$

$$\dot{F}=\partial_0 F,\quad F'=\partial_1 F,\quad \psi^{\dagger}=\left(\psi_{+}^{\dagger}\psi_{-}^{\dagger}\right)$$

and 2 dimensional coordinates x^{μ} $(\mu=0,1)$ are denoted by (t,x) and take $-\infty<x<\infty,\quad -\infty<t<\infty$.

where $\pi_{\pm}^{A} \equiv \partial\mathcal{L}/\partial\dot{A}_{\pm}$. Consistency condition for the primary constraints leads the secondary constraints,

$$\dot{\pi_{\pm}^{A}} = -\left\{H_0, \pi_{\pm}^{A}\right\} = j_{\pm} \approx 0, \tag{2.4}$$

where $j_{\pm} \equiv \psi_{\pm}^{\dagger}\psi_{\pm}$. Here the Hamiltonian H_0 is

$$H_0 = \int dy \left[i\left(\psi_{+}^{\dagger}\psi_{+}{}' - \psi_{+}^{\dagger}\psi_{-}{}'\right) - A_{-}j_{+} - A_{+}j_{-} + \lambda_{+}\pi_{+}^{A} + \lambda_{-}\pi_{-}^{A}\right], \tag{2.5}$$

where $\lambda_{\pm}$ are Lagrange multiplier for the primary constraints. The consistency condition for $j_{\pm}$ is automatically satisfied and all super-Poisson brackets vanish. Therefore all constraints are first-class at classical level. The canonical theory of this system is characterized by the first-class constraints § (2.3) (2.4).

For the quantization of the model ¶ , we replace the super-Poisson bracket by the super-commutator. To regularizing operator products, we adopt the normal ordering prescription. The definition of normal ordering in Appendix B is described. Now the equal-time commutators between operator $j_{\pm}$ does not vanish,

$$\left[j_{\pm}\left(x,t\right), j_{\pm}\left(y,t\right)\right] = \pm\frac{i}{2\pi}\partial_x\delta\left(x-y\right). \tag{2.6}$$

The term in r.h.s. is so-called Schwinger term which represents the commutator anomaly of gauge algebra. The form of this term agrees with a form of the cohomology given in Appendix A. Namely, the form of the Schwinger term is independent of the choice of regularizations and gauge-fixing, except for the overall coefficient $\pm i/2\pi$. The consistency condition for $j_{\pm}$ is given by

$$\left[j_{\pm}\left(x,t\right), H_0\right] = \pm ij_{\pm}{}'\left(x,t\right) \mp \frac{i}{2\pi}A_{\mp}{}'\left(x,t\right). \tag{2.7}$$

To guarantee the consistency condition for $j_{\pm}$, new constraints are imposed as follows

$$A_{\pm}{}'\left(x,t\right) \approx 0. \tag{2.8}$$

Moreover the consistency condition gives "$\lambda_{\pm} = 0$". Because of the anomaly, these constraints become second class all together.

Next, we modify the theory to recover all the classical local symmetries violated by anomalies. We apply the BF formalism to recover the classical local symmetries.

§The variable $\psi_{\pm}^{\dagger}$ is taken as the canonical momentum conjugate to $\psi_{\pm}$.

¶We use $\hbar = 1$ and omit the normal ordering symbol as given in Appendix A. For definitions of the equal-time anti-commutation relations, we simply replace the super-Poisson bracket $\{A, B\}$ by the graded commutator $-i\left[A, B\right]$ defined as $\left[A, B\right] = AB - BA\left(-\right)^{\epsilon(A)\epsilon(B)}$. Where $\epsilon\left(A\right)$ is the grassmannian parity of A. This naive prescription can be justified in the BF formalism, because we do not use the Dirac bracket.

This can be carried out without affecting the physical contents of the original theory, by introducing extra degrees of freedom which can be gauged away by the recovered local symmetry. Following the general idea of BF[12], we introduce a canonical pair of bosonic fields (π_θ, θ), which is called as BF fields henceforth. They are assumed to satisfy the following commutation relation

$$[\theta(x,t), \pi_\theta(y,t)] = -i\delta(x-y). \tag{2.9}$$

Notice that the BF field must have negative metric in this model. The constraint can be modified by adding to $j_\pm$ an appropriate term containing BF field to cancel the Schwinger term in Eq.(2.6). The new modified constraint is given by

$$\tilde{j}_\pm \equiv j_\pm + k(\pi_\theta \pm \theta') \approx 0. \tag{2.10}$$

if $k^2 = 1/4\pi$[‖], then the algebras of modified constraints become

$$\left[\tilde{j}_\pm(x,t), \tilde{j}_\pm(y,t)\right] = \ 0. \tag{2.11}$$

Cancelation of the second term in r.h.s.of Eq.(2.7) also occurs by a modification of the Hamiltonian. The modified Hamiltonian is constructed by adding some polynomials of the BF fields to the Hamiltonian in (2.5). Our choice is expressed by

$$\tilde{H}_0 = H_0 - \int dx[\frac{1}{4}(\pi_\theta + \theta')^2 + \frac{1}{4}(\pi_\theta - \theta')^2]. \tag{2.12}$$

The Eq.(2.10) and Eq.(2.12) become the first class

$$\left[\tilde{j}_\pm(x,t), \tilde{H}_0\right] = \ \pm i\tilde{j}_\pm{}'(x,t). \tag{2.13}$$

Here, the consistency condition of $\tilde{j}_\pm$ is satisfied without imposing new constraints in Eq.(2.8). Therefore owing to the extra degree of freedom π_θ and θ, the original second class system becomes an effectively first class system.

Then, according to the generalized Hamiltonian formalism developed by Batalin, Fradkin and Vilkovisky (BFV formalism) [13][**], we introduce canonical sets of ghosts and anti-ghosts along with the auxiliary fields, for each constraint $\pi^A_\pm \approx 0$ and $\tilde{j}_\pm \approx 0$. In other word, we define the extended phase space (EPS),

$$\tilde{j}_\pm : (\mathcal{C}^\pm, \bar{\mathcal{P}}_\pm), (\mathcal{P}^\pm, \bar{\mathcal{C}}_\pm), (N^\pm, B_\pm);$$

$$\pi^A_\pm : (\mathcal{C}^\pm_\pi, \bar{\mathcal{P}}^\pi_\pm), (\mathcal{P}^\pm_\pi, \bar{\mathcal{C}}^\pi_\pm), (N^\pm_\pi, B^\pi_\pm).$$

‖ If the canonical commutation relation $[\theta(x,t), \pi_\theta(y,t)] = i\delta(x-y)$ is imposed, we have $k^2 = -1/4\pi$, with k being imaginary. But k must be real because of Hermiticity of Lagrangian density.

** For review, see[14]

The off-shell nilpotent BRST charge is given by

$$\begin{aligned} Q &= Q_- + Q_+; \\ Q_\pm &\equiv \int dx \left[\mathcal{C}^\pm \tilde{j}_\pm + \mathcal{C}^\pm_\pi \pi^A_\pm + \mathcal{P}^\pm B_\pm + \mathcal{P}^\pm_\pi B^\pi_\pm \right] \end{aligned} \tag{2.14}$$

This charge Q generates the BRST transformation of the fundamental variables.

$$\begin{array}{llllllll} \delta\psi_\pm = & -i\mathcal{C}^\pm\psi_\pm, & \delta\psi^\dagger_\pm = & i\mathcal{C}^\pm\psi^\dagger_\pm, & \delta A_\pm = & \mathcal{C}^\pm_\pi, & \delta\mathcal{C}^\pm_\pi = & 0, \\ \delta N^\pm = & \mathcal{P}^\pm, & \delta\mathcal{P}^\pm = & 0, & \delta N^\pm_\pi = & \mathcal{P}^\pm_\pi, & \delta\mathcal{P}^\pm_\pi = & 0, \\ \delta\bar{\mathcal{C}}_\pm = & -B_\pm, & \delta B_\pm = & 0, & \delta\bar{\mathcal{C}}^\pi_\pm = & -B^\pi_\pm, & \delta B^\pi_\pm = & 0, \\ \delta\bar{\mathcal{P}}_\pm = & -i\tilde{j}_\pm, & \delta\mathcal{C}^\pm = & 0, & \delta\bar{\mathcal{P}}^\pi_\pm = & -\pi^A_\pm & \delta\pi^A_\pm = & 0, \\ \delta\theta = & -k\left(\mathcal{C}^+ + \mathcal{C}^-\right), & \delta\pi_\theta = & -k\left(\mathcal{C}^+ - \mathcal{C}^-\right)', & & & & \end{array} \tag{2.15}$$

and the BRST invariant Hamiltonian, which is called as the minimal Hamiltonian,

$$H_{min} = \tilde{H}_0 + \int dx \left(\overline{\mathcal{P}}_+ \mathcal{C}^{+\prime} - \overline{\mathcal{P}}_- \mathcal{C}^{-\prime} \right). \tag{2.16}$$

The second term in (2.16) is required for the minimal Hamiltonian to respect BRST invariance.

The dynamics of the system are controlled by the BRST invariant total Hamiltonian H_T, which consists of the minimal Hamiltonian and the gauge-fixing term. In the present case, H_T is given by

$$H_T = H_{min} + \frac{1}{i}\left[Q, \Psi\right]. \tag{2.17}$$

where Ψ is a gauge fermion. The standard form of Ψ is given by

$$\begin{aligned} \Psi &= \Psi_+ + \Psi_-; \\ \Psi_\pm &\equiv \int dx \left[\overline{\mathcal{C}}_\pm \chi^\pm + \overline{\mathcal{C}}^\pi_\pm \chi^\pm_\pi + \overline{\mathcal{P}}_\pm N^\pm + \overline{\mathcal{P}}^\pi_\pm N^\pm_\pi \right], \end{aligned} \tag{2.18}$$

where $\chi^\pm, \chi^\pm_\pi$ denote the gauge conditions imposed on dynamical variables. ††

†† To construct the effective action, we choose the standard form of gauge fermion shifted by $\Psi_\pm \to \Psi_\pm + \int dx \left(\overline{\mathcal{C}}_\pm \dot{N}^\pm + \overline{\mathcal{C}}^\pi_\pm \dot{N}^\pm_\pi \right)$.

This just cancels the Legendre terms $\int dx \left[\overline{\mathcal{C}}_\pm \dot{\mathcal{P}}^\pm + \overline{\mathcal{C}}^\pi_\pm \dot{\mathcal{P}}^\pm_\pi + B_\pm \dot{N}^\pm + B^\pi_\pm \dot{N}^\pm_\pi \right]$ in the constructed effective action.

The BRST invariant effective action can be obtained as

$$
\begin{aligned}
S_{eff} &= \int d^2x \left[i\psi_\pm^\dagger \dot{\psi}_\pm + \pi_\pm^A \dot{A}_\pm + \bar{\mathcal{P}}_\pm \dot{\mathcal{C}}_\pi^\pm + \bar{\mathcal{P}}_\pm^\pi \dot{\mathcal{C}}_\pi^\pm - \pi_\theta \dot{\theta} \right] - \int dt H_T \\
&= \int dxdt \Big[\psi_+^\dagger \left(i\partial_- - N^+ \right) \psi_+ + \psi_-^\dagger \left(i\partial_+ - N^- \right) \psi_- \\
&\qquad -\pi_\theta \dot{\theta} + \tfrac{1}{2} \left(\pi_\theta^2 + \theta'^2 \right) - k \left(\pi_\theta + \theta' \right) N^+ - k \left(\pi_\theta - \theta' \right) N^- \\
&\qquad +\pi_+^A \left(\dot{A}_+ - N_\pi^+ \right) + \pi_-^A \left(\dot{A}_- - N_\pi^- \right) \\
&\qquad +\bar{\mathcal{P}}_+ \left(\partial_- \mathcal{C}^+ - \mathcal{P}^+ \right) + \bar{\mathcal{P}}_- \left(\partial_+ \mathcal{C}^- - \mathcal{P}^- \right) + \bar{\mathcal{P}}_\pi^\pm \left(\dot{\mathcal{C}}_\pi^\pm - \mathcal{P}_\pi^\pm \right) \\
&\qquad -\left(B_\pm \chi^\pm + B_\pm^\pi \chi_\pm^\pi + \bar{\mathcal{C}}_\pm^\pi \delta \chi_\pm^\pi + \bar{\mathcal{C}}_\pm \delta \chi^\pm \right) \Big].
\end{aligned}
\tag{2.19}
$$

Since the auxiliary fields $N^\pm$ play the roles of $-A_\mp$ in Eq.(2.19), respectively, The following gauge conditions are imposed,

$$
\chi_\pm^\pi = N^\pm + A_\mp. \tag{2.20}
$$

This gauge fixing condition is essential to geometrize the gauge fields. As for the rest of the gauge conditions $\chi_\pm$, we assume that they and their BRST transformations are independent of, $\pi_\pm^A, \pi_\theta, B_\pm^\pi, \bar{\mathcal{P}}_\pm, \bar{\mathcal{P}}_\pm^\pi, \bar{\mathcal{C}}_\pm^\pi$. By taking the variation of Eq.(2.19) with respect to variables, we can derive equations of motion for $\pi_\pm^A, \bar{\mathcal{P}}_\pm, \bar{\mathcal{P}}_\pm^\pi, B_\pm^\pi, \bar{\mathcal{C}}_\pm^\pi$ and π_θ,

$$
\begin{aligned}
&\dot{A}_\pm = N_\pi^\pm, \quad \partial_\mp \mathcal{C}^\pm = \mathcal{P}^\pm, \quad \dot{\mathcal{C}}_\pi^\pm = \mathcal{P}_\pi^\pm, \quad N^\pm = -A_\mp, \\
&\mathcal{P}^\pm = -\mathcal{C}_\pi^\mp, \quad \dot{\theta} - \pi_\theta - k \left(A_- + A_+ \right) = 0.
\end{aligned}
\tag{2.21}
$$

The BRST transformations in the configuration space is given by replacing the parameters of infinitesimal gauge transformation with ghost fields. There exist two independent ghost variables C_A, C_V in the configuration space, which correspond to $U(1)_A$ symmetry and the $U(1)_V$ symmetry respectively. After imposing the gauge condition in Eq.(2.20), we obtain the geometrization for the gauge field and the ghost variables in EPS as

$$
\partial_\mp \mathcal{C}^\pm = -\mathcal{C}_\pi^\mp, \tag{2.22}
$$

And the BRST transformation for $A_\pm$ in the EPS is given by

$$
\delta A_\pm = -\partial_\pm \mathcal{C}^\mp. \tag{2.23}
$$

The BRST transformations for $\psi_\pm$ and $A_\pm$ in configuration space are expressed as follows,

$$
\delta \psi_\pm = \pm i \left(C_A + C_V \right) \psi_\pm, \qquad \delta A_\pm = \mp \partial_\pm \left(C_A \mp C_V \right). \tag{2.24}
$$

By requiring the equality between the BRST transformation in EPS and the one in configuration space, we get the following relation

$$\mathcal{C}^{\pm} = \mp \left(C_A \pm C_V \right). \tag{2.25}$$

The effective action Eq.(2.19) contains many non propagating fields, i.e. $\pi_{\pm}^{A}$, $\mathcal{P}_{\pi}^{\pm}$, $\bar{\mathcal{P}}_{\pm}$, $\bar{\mathcal{P}}_{\pm}^{\pi}$, $N^{\pm}$,$N_{\pi}^{\pm}$, $\pi_{\pm}^{A}$ and π_{θ}, which can be eliminated by virtue of the equations of motion Eq.(2.21). After eliminating these variables from the master action, We arrive at the following BRST invariant effective action in the most general class of gauges.

$$\begin{aligned} S_{eff} \;\; = \;\; \int dxdt : \Big[& \psi_{+}^{\dagger} \left(i\partial_{-} + A_{-} \right) \psi_{+} + \psi_{-}^{\dagger} \left(i\partial_{+} + A_{+} \right) \psi_{-} \\ & - \frac{1}{2} \partial_{+}\theta \partial_{-}\theta + k \left(A_{-}\partial_{+}\theta + A_{+}\partial_{-}\theta \right) - \frac{1}{4\pi} A_{-} A_{+} \\ & - \frac{1}{8\pi} \left(A_{+}^{2} + A_{-}^{2} \right) \\ & - \left(B_{+}\chi^{+} + \bar{\mathcal{C}}_{+}\delta\chi^{+} + B_{-}\chi^{-} + \bar{\mathcal{C}}_{-}\delta\chi^{-} \right) \Big] : . \end{aligned} \tag{2.26}$$

The effective action contains two types of counter term. One is covariant type, which is constructed by the gauge fields and BF fields. The other is non-covariant type, which is constructed by only the gauge fields. The origin of appearance of non-covariant term is related to the fact, that the manifest two dimensional covariance is violated in the class of regularization schemes as we prove in Appendix B. In order to recover the two dimensional covariance, we need an appropriately chosen noncovariant counter term. It is nothing but this counter term. The covariant counter term corresponds to one in the Thirring model coupled to the gauge fields[10].

In this section, we investigated the model as anomalous gauge theory by applying the generalized Hamiltonian formalism of BF and BFV, and got BRST invariant effective action. This enables us to formulate the theory in most general class of gauges.

3 Explicit gauge fixed effective action

3.1 conformal gauge

In the previous section we have formulated the BRST invariant effective action. So far our argument did not rely on particular gauge condition; the effective action can be applicable to any gauge fixing. In this section, we will investigate the explicit gauge fixed theory.

First, we will consider the gauge fixed theory under $A_{\pm} = 0$. As the result, we show that the physical excitation mode consists of only a null state as asserted in [6].

We have recovered the local $U(1)_V \times U(1)_A$ symmetries employing the BF formalism at quantum level. Therefore we can choose the following gauge fixing condition

$$\chi_\pm = A_\pm. \tag{3.1}$$

The gauge current of $A_\pm$ is given by $J_\pm \equiv j_\pm + k\partial_\pm\theta$ in this gauge.

Integrating out the multipliers $B_\pm$ and eliminating non propagating variables by the equations of motion, we can reduce the effective action to the following expression

$$S_{eff} = \int dxdt \left[\psi_+^\dagger i\partial_-\psi_+ + \psi_-^\dagger i\partial_+\psi_- - \frac{1}{2}\partial_+\theta\partial_-\theta + \bar{\mathcal{C}}_+\partial_-\mathcal{C}^+ + \bar{\mathcal{C}}_-\partial_+\mathcal{C}^-\right]. \tag{3.2}$$

BRST charge is given by

$$\begin{gathered} Q = Q_+ + Q_-, \quad \text{where } Q_\pm = \int dx\mathcal{C}^\pm J_\pm, \\ [Q_+, Q_-] = 0, \quad Q_\pm^2 = 0. \end{gathered} \tag{3.3}$$

BRST invariant stress tensor $T_{\pm\pm}$ is constructed by the Noether's theorem and algebra of stress tensor is $c = 0$ Virasoro algebra. Moreover, $T_{\pm\pm}$ is written in the BRST trivial form

$$T_{\pm\pm}(x,t) = [Q_\pm, X_\pm(x,t)], \tag{3.4}$$

where $X_\pm(y,t) = -i/4\pi\bar{\mathcal{C}}_\pm\left(\psi_\pm^\dagger\psi_\pm - k\partial_\pm\theta\right)$.

The fact in Eq.(3.4) means that there is no physical excitation state of the system. [6]

3.2 light-cone gauge

In this subsection, we will show that a degree of freedom of gauge field becomes a dynamical variable in the light-cone gauge because of gauge anomaly at quantum level.

In the gauge fixing condition

$$\chi^- = A_-, \quad \chi^+ = \frac{\theta}{k}, \tag{3.5}$$

the gauge current of A_- is given by $J_+ = j_+ - k^2 A_+$. The effective action is expressed by

$$S_{eff} = \int d^2x \left[\psi_+^\dagger i\partial_-\psi_+ + \psi_-^\dagger i\partial_+\psi_- + A_+ j_- - \frac{1}{8\pi}A_+^2 - \bar{\mathcal{C}}_-\partial_- C_A\right], \tag{3.6}$$

and the off-shell nilpotent BRST charge is given by

$$Q = -\int dx C_A J_+. \tag{3.7}$$

Q is constructed by the fields, and depends only on x^+. By using the equation of motion $\partial_- \bar{\mathcal{C}}_- = 0$ and the BRST transformation for $\bar{\mathcal{C}}_-$, the equation of motion for the gauge field A_+ is given by

$$\partial_- A_+ = 0. \tag{3.8}$$

The Eq.(3.8) corresponds to the curvature equation in two dimensional gravity.

The commutator for A_+ is given by

$$[A_+ (x), A_+ (y)] = -8i\pi \partial_x \delta (x - y). \tag{3.9}$$

as showed in Appendix D.

In this gauge, the stress tensor based on the Noether's theorem does not generate the translation for fields. However we can construct the true stress tensor as follows, which generates the translation for the fields,

$$\tilde{T}_{++} =: \frac{1}{2} \left(\psi_+^\dagger \partial_+ \psi_+ - \partial_+ \psi_+^\dagger \psi_+ \right) - \bar{\mathcal{C}}_- \partial_+ C_A + \frac{1}{16\pi^2} A_+^2 : . \tag{3.10}$$

Here we have added the contribution of A_+,

$$\frac{1}{16\pi^2} : A_+^2 : \tag{3.11}$$

into the stress tensor based on the Noether's theorem by hand. $\tilde{T}_{++}$ satisfies the Virasoro algebra with $c = 0$ and it is expressed by the following BRST trivial form.

$$\tilde{T}_{++} (x) = i\pi [Q, X (x)], \tag{3.12}$$

where $X (x) = \bar{\mathcal{C}}_- (j_+ + 1/4\pi A_+)$.

The stress tensor based on the Noether's theorem was not true stress tensor at quantum level, because it did not generate any translation for fields. So we modified the stress tensor in analogy to Sugawara construction. One can interpret Eq.(3.11) as the Sugawara form for the gauge field A_+ which satisfies the Kac-Moody algebra as Eq.(3.9). Moreover, $\tilde{T}_{++}$ satisfies the Virasoro algebra with $c = 0$, and it generates the translation for fields at quantum level. Hence, $\tilde{T}_{++}$ is regarded as the true BRST invariant stress tensor. As the result, we recognize that the gauge field A_+ is a dynamical variable at quantum level.

4 Conclusion

In this paper, we investigated the massless Schwinger model without kinetic terms of gauge field as an anomalous gauge theory by applying the generalized Hamiltonian

formalism of the BF and the BFV. The BFV formalism enables us to formulate the theory in most general class of gauge fixing conditions.

Here, we describe the difference between our quantization and the quantization in [6]. In any anomalous gauge theory, the introduction of the Wess-Zumino scalar is independent of the freedom of the gauge fields. The BF field which has be introduced in section 2 corresponds to the Wess-Zumino scalar in the conformal gauge. Owing to the extra degree of freedom, the symmetry violated by anomaly recovered even at quantum level. Therefore we were able to choose the conformal gauge fixing condition $A_{\pm} = 0$ in this system. In the conformal gauge, the physical state is only null state as asserted in [6]. However the property of the author's gauge fixing condition is completely different from our one. The authors parameterize A_+ in term of $U(1)$ group element ϕ which behaves as the Wess-Zumino scalar finally

$$A_+ = -\partial_+ \phi.$$

However, in our formulation, the gauge field A_+ is completely independent of the Wess-Zumino scalar θ. The behavior of the gauge field A_+ as the dynamical variable is derived based on the equation of motion for the anti-ghost.

Our assertion is as follows. If one consider g_{++} instead of A_+, one can understand that this model has the non-critical string like property as follows. In the Light-cone gauge fixed two dimensional gravity, the gravity field g_{++} becomes a dynamical variable because of the conformal anomaly, though all degree of freedom of gravity field are decoupled due to three local symmetries, Weyl symmetry and reparametrization invariance at classical level. On the other hand, in the Schwinger model without the kinetic term of gauge field, the gauge field A_+ becomes a dynamical variable because of the gauge anomaly, though all degree of freedom of gauge field are decoupled due to two local symmetries, $U(1)_V \times U(1)_A$. Therefore we expect that the study of the gauge theory without kinetic term may be useful to understand the non-critical string theory.

Acknowledgments

We thank T.Fujiwara and Y.Igarashi for illuminating discussions, N.Sakai for helpful discussions, for reading the manuscript. We are also grateful to C.Itoi for explaining their paper and his comment, S.Ding for reading the manuscript.

Appendix

A Alternative approach to BRST anomalies

General gauge independent cohomological discussion could be used for searching the anomalies [15]. In this Appendix, we review the general analysis of the BRST anomalies, and apply to this model.

In general, BRST charge Q and total Hamiltonian H_T obey classical gauge algebra. BRST charge is constructed under the nilpotent condition,

$$\{Q, Q\} \quad = \quad 0, \tag{A.1}$$

which means that the constraint which generate classical symmetry is first class constraint.The consistency condition means that BRST charge is time independent, or Hamiltonian is BRST invariant,

$$\{H_T, Q\} \quad = \quad 0. \tag{A.2}$$

At the quantum level, super-Poisson brackets must be replaced with super-commutator s and operator must be regularized by the appropriate prescription(e.g. normal ordering). But we do not suppose specific regularization prescription in this Appendix; so we can discuss the BRST anomalies in the regularization independent way. If there are anomalies in the system, Eq.(A.1) and Eq.(A.2) are broken at quantum level. The anomalous terms may be expanded in $\hbar$ as

$$\begin{aligned} [Q, Q] &\equiv i\hbar\Omega + O(\hbar^3), \\ [Q, H_T] &\equiv i\hbar\Gamma + O(\hbar^3). \end{aligned} \tag{A.3}$$

We assume super-commutator for the operator must obey the commutation law

$$[A, B] = -(-)^{\epsilon(A)\epsilon(B)}[B, A], \tag{A.4}$$

to which the distribution law

$$[A, B + C] = [A, B] + [A, C], \tag{A.5}$$

and the super-Jacobi identity

$$(-)^{\epsilon(A)\epsilon(B)}\,[A, [B, C]] + (-)^{\epsilon(B)\epsilon(C)}\,[B, [C, A]] + (-)^{\epsilon(C)\epsilon(A)}\,[C, [A, B]] = 0 \tag{A.6}$$

hold. By these identity, we easily find

$$\begin{aligned} [Q, [Q, Q]] &= 0, \\ 2\,[Q, [Q, H_T]] + [H_T, [Q, Q]] &= 0. \end{aligned} \tag{A.7}$$

To the lowest $\hbar^2$ oder, super-Jacobi identities of Q and H_T can be truncated into the two conditions,

$$\delta\Omega = 0, \tag{A.8}$$

$$\delta\Gamma = -\dot{\Omega}, \tag{A.9}$$

where δ is classical BRST transformation. Eq.(A.8) is called Wess-Zumino consistency condition[16], and Eq.(A.9) is called descent equation[17]. We will find Ω in the exhaustive fashion, and construct the most general solution of Wess-Zumino consistency condition in regularization independent way.

If Ω is a solution of (A.8), then $\tilde{\Omega} \equiv \Omega + \delta\Xi$ is a solution too. If we redefine the BRST charge as

$$\tilde{Q} \equiv Q - \frac{\hbar}{2}\Xi. \tag{A.10}$$

The consistency condition is

$$\left[\tilde{Q}, \tilde{Q}\right] \equiv i\hbar^2\Omega \tag{A.11}$$

We are interested in Ω which can not be expressed by Ξ. The Ω is true anomaly.

We will search local form of anomalies,

$$\Omega = \int dx\omega. \tag{A.12}$$

In this case Putting $C \equiv dim\mathcal{C}^{\pm}$, we find

$$dim\mathcal{C}^{\pm}_{\pi} = C + 1, \quad dim\overline{\mathcal{P}}_{\pm} = 1 - C, \quad dim\overline{\mathcal{P}}^{\pi}_{\pm} = -C. \tag{A.13}$$

So, $dimQ = C$, $ghQ = 1$ and $dim\Omega = 2C$, $gh\Omega = 2$. If ω is a solution of Wess-Zumino consistency condition Eq.(A.8), then

$$\tilde{\omega} \equiv \omega + \delta h_1 + \partial_1 h_2 \tag{A.14}$$

is a solution too. We are interested in ω which can not be expressed by δh_1 , ∂h_2. The ω is true anomaly. Here $dim\omega = 2C + 1$,$gh\omega = 2$,

	h_1	h_2
dim	$C+1$	$2C$
gh#	1	2

(A.15)

Using information of geometrization in Eq.(2.20) and BRST transformation in Eq.(2.16), we get following table

$$
\begin{array}{lcccccc}
 & (A_{\pm} & , \Pi^{A}_{\pm}) & (\psi_{\pm} & , i\psi^{\dagger}_{\pm}) & & \\
\text{dim} & 1 & 0 & 1/2 & 1/2 & & \\
\text{gh\#} & 0 & 0 & 0 & 0 & & \\
 & (\mathcal{C}^{\pm} & , \overline{\mathcal{P}}_{\pm}) & (\mathcal{P}^{\pm} & , \overline{\mathcal{C}}^{\pm}) & (N^{\pm} & , B_{\pm}) \\
\text{dim} & C & 1-C & C+1 & -C & 1 & 0 \\
\text{gh\#} & 1 & -1 & 1 & -1 & 0 & 0 \\
 & (\mathcal{C}^{\pm}_{\pi} & , \overline{\mathcal{P}}^{\pi}_{\pm}) & (\mathcal{P}^{\pm}_{\pi} & , \overline{\mathcal{C}}^{\pm}_{\pi}) & (N^{\pm}_{\pi} & , B^{\pi}_{\pm}) \\
\text{dim} & C+1 & -C & C+2 & -C-1 & 2 & -1 \\
\text{gh\#} & 1 & -1 & 1 & -1 & 0 & 0
\end{array}
\tag{A.16}
$$

We divide EPS into two sectors

$$
\begin{aligned}
S_1 &\ : \ \text{consisting of} \quad \left(\psi^{\dagger}_{\pm}, \psi_{\pm}\right) \quad \left(\mathcal{C}^{\pm}, \overline{\mathcal{P}}_{\pm}\right) \\
S_2 &\ : \ \text{consisting of all the other fields}
\end{aligned}
$$

The each sector is orthogonal to other sector. In other words, δ operation of the each sector satisfies

$$\delta_1^2 = \delta_2^2 = 0, \qquad \delta_1\delta_2 + \delta_2\delta_1 = 0, \tag{A.17}$$

where $\delta = \delta_1 + \delta_2$. The S_2 sector is BRST trivial, and there is no non-trivial solution. And we assume that the ω preserve the invariance under the following global discrete symmetry*,

$$\pm \rightarrow \mp, \quad \partial_x \rightarrow -\partial_x. \tag{A.18}$$

Therefore the true anomaly ω is given by

$$\omega = \kappa \left(\mathcal{C}^{+}\partial_1\mathcal{C}^{+} - \mathcal{C}^{-}\partial_1\mathcal{C}^{-}\right). \tag{A.19}$$

We find that ω is not represented by δh_1 and $\partial_1 h_2$. And we will find $\Gamma = \int dx\gamma$,

$$\gamma = -4\kappa \left(N^{+}\partial_1\mathcal{C}^{+} - N^{-}\partial_1\mathcal{C}^{-}\right). \tag{A.20}$$

This local form of Γ satisfies descent equation Eq.(A.9).

*If we take care of the result of geometrization , $\partial_{\mp}\mathcal{C}^{\pm} = -\mathcal{C}^{\mp}_{\pi}$, then $\partial_x \rightarrow -\partial_x$ is imposed

B The definition of the normal ordering

The overall coefficient κ in Eq.(A.19) or Eq.(A.20), however, remains undetermined in the algebraic method in Appendix A. To fix κ, we must define operator products by some ordering prescription, and then examine the nilpotency of the BRST charge. For the purpose to define operator ordering, we decompose operators into parts by

$$F^{(\pm)}(x) = \int dy \delta^{(\mp)}(x-y) F(y) \ ; \text{ for } F = \psi_+, \psi_+^\dagger, \bar{\mathcal{C}}_+, \mathcal{C}_+, \pi_\theta + \theta'$$
$$F^{(\pm)}(x) = \int dy \delta^{(\pm)}(x-y) F(y) \ ; \text{ for } F = \psi_-, \psi_-^\dagger, \bar{\mathcal{C}}_-, \mathcal{C}_-, \pi_\theta - \theta'$$

where $\delta^{(\pm)}(x) = \pm i/2\pi (x \pm i\epsilon)$. The $F^{(+)}$ and $F^{(-)}$ thus defined reduce, respectively, to positive- and negative-frequency part in the conformal gauge. By putting $F^{(+)}$'s to the right of $F^{(-)}$'s, we define operator ordering. We thus obtain

$$\kappa^2 = 1/4\pi$$

for the anomaly coefficient.

C The derivation of the commutator for A_+

The BRST transformation of A_+, which has been read off from Eq.(2.16) and the variation for the anti-ghost is given by

$$\delta A_+ = \partial_+ C_A. \tag{C.1}$$

On the other hand, δA_+ is defined by

$$\delta A_+(x^+) = -i[A_+(x^+), Q_A] \quad \text{where} \quad Q_A = \frac{1}{4\pi}\int dx^+ C_A A_+. \tag{C.2}$$

Comparing this with δA_+ given in Eq.(C.1), we obtain the commutation relation Eq.(3.10).

D Quantum BRST algebra in conformal gauge

In this subsection, we will discuss the quantum BRST algebra of the system. It might be another approach to construct quantum topological field theory.

In the conformal gauge, the BRST charge is given by

$$Q = Q_+ + Q_-, \quad \text{where} \quad Q_\pm = \int dx \mathcal{C}^\pm (j_\pm + k\partial_\pm \theta). \tag{D.1}$$

We define anti-BRST charge [18] using uncertain sign of the level k.

$$\overline{Q} = \overline{Q}_+ + \overline{Q}_-, \quad \text{where} \quad \overline{Q}_\pm = \int dx \mathcal{C}^\pm (j_\pm - k\partial_\pm \theta). \tag{D.2}$$

Ghost number charge is

$$Q_c = \int dx \left[\overline{\mathcal{C}}_+ \mathcal{C}^+ + \overline{\mathcal{C}}_- \mathcal{C}^- \right]. \tag{D.3}$$

The quantum extended BRST algebra is

$$\begin{aligned}
[\, Q, Q \,] &= 0, \qquad [\, \overline{Q}, \overline{Q} \,] = 0, \\
[\, iQ_c, Q \,] &= +Q, \qquad [\, iQ_c, \overline{Q} \,] = -\overline{Q}, \\
[\, Q_c, Q_c \,] &= 0, \\
[\, Q_\pm, \overline{Q}_\pm \,] &= 4\pi i \int dx T_{\pm\pm}.
\end{aligned} \tag{D.4}$$

The extended BRST algebra and its representation has been well studied in classical level [19]. The quantum extended BRST algebra was studied in the flame of conformal field theory. This algebra correspond to $c = 0$ trivial topological field theory discussed in [20], and this model give dynamical representation of $c = 0$ trivial topological field theory.

References

[1] E.Guadagnini,M.Martellini,and M.Mintchev, *Phys.Lett.* **194B**(1987)69;
K.Bardakci,E.Rabinovici,and B.Saring, *Nucl. Phys.* **B299**(1988) 151;
K.Gawedzki and A.Kupianinen, *Phys. Lett.***215B**(1988)119;
P.Bowcock, *Nucl.Phys.***B316**(1989)80;
A.N.Redlich and H.J.Schnitzer, *Phys.Lett.***193B**(1987)471;
H.J.Schnitzer, *Nucl.Phys.***B324**(1989)412.

[2] A.M.Polyakov and P.B.Wiegmann, *Phys.Lett.***131B**(1983)121;**141B**(1984)223;
A.M.Polyakov,Lectures given at Les Houches Summer School,1988.

[3] D.Karabali and H.J.Schnitzer, *Nucl.Phys.***B329**(1990)649

[4] Y.Tanii, *Mod.Phys.Lett.***A5**(1990)927

[5] P.Goddard,A.Kent and D.Olive,*Phys.Lett.***152**(1985)88;
*Comm.Math.Phys.***103**(1986)105

[6] C.Itoi and H.Mukaida, *Mod.Phys.Lett.***A7**(1992)259

[7] E.Witten. *Commoun.Math.Phys.***117**(1988)353

[8] J.Schwinger, *Phys.Rev*(1990)2425

[9] A.M.Polyakov,*Mod.Phys.Lett.***A2**(1987)893

[10] Y.Watabiki, *Phys.Rev.***D40**,(1989)1229

[11] P.A.M.Dirac, Lecture on Quantum Mechanics,Yeshiba Univ.Press,
*Pro. R. Soc.***A246**(1958)326

[12] I.A.Batalin, E.S.Fradkin, *Phys.Lett.***180B**(1986)157;

[13] E.S.Fradkin and G.A.Vilkovisky, *Phys.Lett.***55B** (1975)224;
I.A.Batalin and G.A.Vilkovisky, *Phys.Lette.***69B**(1977)309

[14] M.Henneaux, *Phys.Rep.***126**(1985)1;
I.A.Batalin, E.S.Fradkin, *Riv.Nuovo.Cimento***9**(1986)1;
*Ann. Inst. Henri. Poincaré,***49**(1989)145

[15] T.Fujiwara, Y.Igarashi and J.Kubo, *Nucl.Phys.***B358** (1991)195;
T.Fujiwara, Y.Igarashi, J.Kubo, and K.Maeda, *Nucl.Phys.***B391** (1993)211;
T.Fujiwara, Y.Igarashi, M.Koseki, R.Kuriki and T.Tabei, *Nucl.Phys.*
B425(1994)289

[16] J.Wess and B.Zumino, *Phys.Lett.***B37**(1971)95

[17] B.Zumino, in Relativity,groups and topology II,Les Houches 1983
ed. B.S.DeWitt and R.Stora (North-Holland,Amsterdam, 1984)

[18] G.Curci and R.Ferrari, *Phy.Lett***63B**(1976)91;
I.Ojima, *Prog.Theor.Phys.***64**(1980)625

[19] K.Nishijima, *Nucl.Phys.***B238**(1984)601;*Prog.Theor.Phys.***73**(1985)536

[20] T.Eguchi and S.K.Yang, *Mod.Phys.Lett.***A5**(1990)1693